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Prime Numbers: Enigma Variations and Arthur Koestler's Window on Infinity*

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From antiquity to the present there has been continual, often intense, interest in the category of mathematics known as "number theory," which fascinates us with conjectures about relationships between or among whole numbers (this may include negative numbers).

Numbers help us to understand infinity. Let's begin with some basic terminology and then move on to probe the relationships of whole numbers to each other before investigating those very special numbers known as "primes." Once that investigation is completed we can conclude by noting some oddities of, and some practical uses for, these maverick prime numbers.

Mathematics has special terms for types of numbers: some are familiar (amicable, cardinal, ordinal, abundant); others refined (complex, composite, natural, perfect, prime, twin), others subtly hostile (negative; irrational, radical), some are purely whimsical (imaginary, deficient), others intimidating (algebraic, titanic) or even shapely (round, square, oblong, triangular), and some bizarre or eccentric (surreal, transfinite, pronic, transcendental).

The rarefied realm of "prime" numbers has engendered its own specific vocabulary. There are "gigantic," "primorial," "repunit," "titanic" and "twin" primes, all of which can be the subject of paragraphs if not pages in mathematical textbooks and treatises. In this essay the focus is on “infinitesimal" primes, numbers so large that they can best be expressed only as multiples raised to the nth power plus or minus 1. That apparently limitless dimension of primes was perhaps best defined by a journalist rather than a mathematician (as we'll see below) during the dangerous years of the Spanish Civil War (1936-1939).

Prime numbers are those (such as 2, 3, 7, 11, 17, 23, 29 etc.) divisible only by themselves and one, and thus differentiated from composite numbers (all others). Put another way, primes are "self-generated" integers, none of which exists via multiples of smaller numbers ("1" was once considered to be a prime, but is categorized more recently as neither prime nor composite). Due to that quality, primes elicited attention as far back in time as the mathematician Euclid c. 300 B.C., who included a formula for determining prime numbers in *Elements*, his famous textbook on math (Toomer, 1996: 564). For a poetic tribute to Euclid, see Appendix 1 of this essay.

That formula (so far not disproved) indicates that primes are infinite. Though their occurrence diminishes as numerical sequence increases, one can (argued Euclid) always find a prime beyond that number which *appears to be* the last and largest of these mavericks. About a century later (c. 200 B.C.) the polymath Eratosthenes (Fraser, 1996: 553-554) devised a simple and still usable method ("Eratosthenes' sieve"–see Conway & Guy, 1996: 129 Fig. 5.1 for a diagram) for separating primes from composite numbers. For Greek thinkers, the *theorem* of understanding the property of numbers was as important as any practical value that ensued. They worked without computers and with few library resources in the field of mathematics. Clawson (1996:147 Fig. 34) provides a chart of the prime numbers between 1 and 100, a tribute to early Greek mathematics and a microcosm of the vast realm in which they reside.

The problem with huge numbers (those having hundreds or even thousands of digits) is determining which are and are not, primes. Until fairly recently, computational time to identify large primes was exceedingly high. Thus the announcement that three mathematicians--a professor and two students at the Indian Institute of Technology in Kanpur--had devised a rapid computer system for isolating primes caused quite a stir in mathematical circles worldwide (Johnson, 2002–see also Appendix 2 of this essay).

In spite of that breakthrough, there are those less concerned with the "outer limits" of prime numbers than with the "patterns" they make when arranged in numerical squares of various sizes--the larger the square, the easier it is to see that "patterning" where primes tend to align themselves in diagonal or rectilinear fashion, or simply form a cluster.

Some properties of primes seem more like *enigma variations* (the term is from the suite of musical profiles of unidentified friends by early 20th century British composer Edward Elgar). For instance, the integer "2" is the only known *even* prime number; some primes occur in patterns of "twins": two primes, such as 3 and 5, or 5 and 7, or 11 and 13, that are separated by only a single composite number.

But that would be to distract us from the main fascination of primes: their ability to help us conceptualize *infinity*. Euclid's theorem indicates that the number of primes, just like the number of integers, is infinite. It is still uncertain whether those twin primes, occurring (apparently) randomly, irregularly, erratically, are infinite as well. For all their welcome enthusiasm regarding the "magic" of prime numbers, essays, feature articles and books generally fail to mention their equally powerful ability to evoke a sense of wonder, a hint of the majesty of an infinite far beyond the everyday sense of self, what Arthur Koestler referred to (see below) as "a fragrance of eternity."

There is no better example of that almost "numinous" aspect of primes than the personal experience--mystical or religious or even delusional--described by Hungarian-born writer Arthur Koestler (1905-1983). The fullest account of that appears in the second volume of Koestler's autobiography, *The Invisible Writing* (Koestler, 1954; Danube edition 1969). While it's too lengthy to reproduce in its entirety here, a few excerpts may entice readers to consult the entire episode.

Some preliminary notes will provide a context. Koestler worked as a correspondent for several European newspapers, no more notably so than for the liberal, anti-Fascist *News Chronicle* of Britain in 1936-37. His assignment was to report the Spanish Civil War and to that end Koestler, from the front, proved himself a seasoned and savvy reporter with fifteen years' experience of international and European affairs. Nevertheless his luck ran out following the siege and capture of Malaga by Francisco Franco's forces in early 1937.

Koestler, widely known to be a member of the Communist Party since 1933, was captured by pro-Franco forces intent on eliminating aliens and Communists. He was jailed, sentenced to death by a court martial, and was awaiting the firing squad (he could hear other prisoners being taken to it) when his knowledge of math allowed him an unexpectedly welcome vision of an eternity beyond his cell:

I was standing at the recessed window and, with a piece of iron spring that I had extracted from the wire mattress, was scratching mathematical formulae on the wall. I was trying to remember how to derive the formula of the hyperbola, and was stumped; then I tried the ellipse and the parabola and to my delight I succeeded. Next I went on to recall Euclid's famous proof that the number of primes is infinite.

Since I had become acquainted with Euclid's proof at school, it had always filled me with a deep satisfaction that was aesthetic rather than intellectual. As I recalled the method and I scratched the symbols on the wall, I felt the same enchantment. And then, for the first time, I suddenly understood the reason for this enchantment: the scribbled symbols on the wall represented one of the rare cases where a meaningful and comprehensive statement regarding the infinite is arrived at by precise and finite means.

The infinite is a mystical mass shrouded in a haze; and yet it was possible to gain some knowledge of it without losing oneself in treacly ambiguities. The significance of this swept over me like a wave. The wave had originated in an articulate verbal insight; but this evaporated at once leaving in its wake only wordless essence, a fragrance of eternity...

I must have stood there for some minutes, entranced, with a mute awareness that "this is perfect, perfect"--until I noticed some slight mental discomfort nagging at the back of my mind, some trivial circumstance that marred the perfection of the moment. Then I recalled the nature of the irrelevant annoyance; I was, of course, in prison and might be shot...

Whether the experience had lasted for a few minutes or an hour I never knew. In the beginning it occurred two or even three times a week, then the intervals became longer. It could never be induced involuntarily. After freedom it recurred at even longer intervals, perhaps once or twice a year. But by that time the groundwork for a change of personality was completed. I shall henceforth refer to these experiences as "the *hours by the window.*" (Koestler 1969: 428-30)

Koestler's "revelation" does not reflect his own religious background (his family was nominally Jewish) nor any beliefs he himself expressed through prolific publications–see MacAdam, 2006: 84-86 for his fascination with metaphysics, coincidence, and the paranormal. Nevertheless this incident left an indelible image in his mind, which--even if we cannot share its immediacy and intensity--he tried to transmit through language. Yet in the recent (and controversial) biography by David Cesarani, *Arthur Koestler: The Homeless Mind*, 2000) this episode is dismissed as "mumbo-jumbo" (Cesarani, 2000: 139-142 at 142).

Perhaps that expression reflects the biographer's own disinterest in either "metaphysics" in general or number theory in particular. Primes can be far more than a hobby or a mental exercise--they can provide both mathematicians and scientists with what Koestler, in his confinement, touched upon. Their practical value is to draw us into contemplation of the visible universe. Is it really *finite*, as some would argue? Is space "curved" so that we should imagine some ultimate limit to it as a cosmic sphere? Or is it really infinite, without dimension and form, shading off eternally to a some farther distance than we (via mathematics, or science) can ever imagine? For a recent, concise analysis of infinity see Wallace, 2004.

Controversial French philosopher and essayist Joseph de Maistre (1753-1821) believed that "The concept of number is the obvious distinction between the beast and man"(Anonymous, 2007). De Maistre went on to note that because of numbers, screams are transformed into songs, noises take on rhythm, movement becomes dance, static force morphs into dynamic energy, and shadows emerge as real figures. Prime numbers take that imagery into yet another dimension: far beyond mere magic, but perhaps just short of a full epiphany.

What Koestler experienced during those "hours by the window" might be characterized as both an outward and an inward journey, lending itself equally to contemplating the vastness of the universe beyond his prison cell and simultaneously exploring the innermost recesses of his own soul. Hopefully the long-awaited and perhaps definitive biography of Koestler, Michael Scammell's *Cosmic Reporter* (to be published in autumn 2009)will give that episode the attention it deserves.

Oddities and Uses of Primes

It is important to distinguish Number Theory from the pseudo-science of "numerology," or an interest in the supposedly "arcane" aspect of numbers. Admittedly, however, even the vocabulary associated with the rational study of numbers borders on the esoteric, as we saw above.

Within the pantheon of "number theory" names like Dirichlet, Eisenstein, Euler, Fermat, Gauss and Goldbach are as familiar and well-respected as the composers Bach, Mozart, Beethoven, Strauss and Mahler are to classical musicians.

Some of those names are recognizable even to those designated "innumerate" (lacking basic math skills) through such distinctive terminology as "Goldbach's Conjecture" or "Fermat's Last Theorem". Their ancient predecessors in mathematics, particularly Euclid and Eratosthenes, are presently held in the same esteem that fans of early rock 'n roll music reserve for Chuck Berry, Fats Domino, Connie Francis and Brenda Lee.

The oddities of the integers I and 2 were already noted above, and the fact that primes can and do occur in pairs or "twins" was also observed (larger "twins" than those mentioned are 55,049 and 55,051). There are other odd features of some primes; a few may be mentioned here. Certain ones are called "palindromes," which means number sequences that read exactly the same from left to right or from right to left (the simplest is 11, but a more complex prime palindrome is 10,301). The largest so far, computed in 1991, is 11,311 digits (coincidentally, itself a non-prime palindrome) in length and has to be expressed as a mathematical formula.

Some primes are named for the person who first discovered the method of computing them. Among the most famous are Sophie Germain (1776-1831) primes, the brainchild of a French woman who was self-taught in math theory. Primes named for her are those which can be doubled and with the addition of 1 produce another prime. Smallest examples would include 2 (which becomes 5) and 3 (which becomes 7). The largest so far has 3,010 digits and wasn't computed until 1993.

That list of oddities can be extended far beyond the limits of this essay; let's close instead with the delightful observations in Calvin C. Clawson's *Mathematical Mysteries* (1996), to which I have already turned for some guidance about the information given above:

There are tetradic, pandigit, and prime-factorial + 1 primes. And there are Cullen, multifactorial, beastly palindrome, as well as anti-palindrome primes. Then add to these the strobogrammatic, subscript, internal repdigit, and elliptic primes. In fact a whole new branch of mathematics is evolving that deals specifically with the attributes of the various kinds of prime numbers (Clawson, 1996: 181).

Lastly we should note a few instances where prime numbers are of purely practical or “scientific” use. *Encryption* is the art of creating a coded message which is difficult if not impossible to decipher. It can be used by libraries and parents to render inaccessible certain categories of Websites; in the realm of espionage (industrial and/or military), it allows for the safe transmission of highly sensitive information from sender to receiver.

Companies (including banks and other economic institutions), as well as governments, commonly use a "public-key cipher system" often built into computer software systems. This newer method of encoding and then decoding critically important data uses computers to find gigantic composite numbers which will factor (divide into multiple components) into *only two prime numbers*. Thus a sender will encode the message via a composite number and the receiver will decode it via the two prime numbers. The element of infinite numbers of primes is the key factor. For a diagram of a public-key encryption system see Clawson, 1996: 197 Fig. 38.

Not everything about prime numbers and codes need be "secret" or cloak-and-dagger or sinister. There is at least one instance in science fiction where knowledge of primes actually helped to foster intergalactic communication and--ultimately--peaceful and positive relations between earthlings and an intelligent presence far beyond our own tiny corner of the universe. Here I'm not thinking of those musical scales or tones used in Steven Spielberg's *Close Encounters of the Third Kind* (1977), though they worked quite effectively. But musical scales are finite and thus repetitive rather than infinite and unique.

In Carl Sagan's novel *Contact* (1985), and the film version of the same name made a dozen years later (see Poundstone, 1999: 375), aliens signal us that they are "out there" by a series of impulses beamed to earth and caught on receivers monitored by members of the group *SETI* (The Search for Extra-Terrestrial Intelligence). At first the electronic pulses are thought to be random "white noise" from interstellar space. Happily it's the female member of the *SETI* team (Jodie Foster in the film version) who realizes that there is a pattern to the sound impulses, and is able to attribute that to the prime numbers of mathematics. Prime numbers thus become a simple device for the transmission that discloses, not disguises, a true cosmic message.

Appendix 1: Euclid Alone Has Looked on Beauty Bare+

Euclid alone has looked on beauty bare.
Let all who prate of Beauty hold their peace,
And lay them prone upon the earth and cease
To ponder on themselves, the while they stare
At nothing, intricately drawn nowhere
In shapes of shifting lineage; let geese
Gabble and hiss, but heroes seek release
From dusty bondage into luminous air.
O blinding hour, O holy, terrible day,
When first the shaft into his vision shone
Of light atomized! Euclid alone
Has looked on beauty bare. Fortunate they
Who, though once only and then but far away,
Have heard her massive sandal set on stone.
Edna St. Vincent Millay (1892-1950)
from *The Harp Weaver* (1922)

Appendix 2: The Largest Known Primes

Readers who enjoy "crunching numbers" beyond the limited scope of this essay but may also fear the esoteric realms of higher math will benefit from a website (**http://primes.utm.edu/largest.html**) aimed at potential prime numerophiles who want to know more. It is devoted to cataloguing the slowly increasing number of primes that can only be identified with the aid of supercomputers. Its title and its table of contents are worth reproducing here:

The Largest Known Primes*: *A Summary
1. Introduction (what are primes?)
2. The Top Ten Record Primes (updated daily)
3. The Complete List of the Largest Known Primes
4. Other Sources of Prime Information
5. Euclid's Proof of the Infinitude of Primes

There is a link on this page to the website of an organization founded in 1996, **GIMPS** (Great Internet Mersenne Prime Search). It announces new discoveries of this category of prime, the type that lends itself easiest to find. As of August, 2008 a 10 million digit Mersenne prime, the 46th in that series, was identified. For a chart-format summary of "The Largest Known Prime by Year: A Brief History," go to **http://primes.utm.edu/notes/by_year.html**.

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